

Regional Mathematical Olympiad-2009

Time: 3 hours

November 29, 2009

Instructions:

- Calculators (in any form) and protractors are forbidden.
- Rulers and compasses are allowed.
- Answer all the questions. Maximum marks: 100.
- Answer to each question should start on a new page. Clearly indicate the question number.

1. Let ABC be a triangle in which $AB = AC$ and let I be its in-centre. Suppose $BC = AB + AI$. Find $\angle BAC$. [16]
2. Show that there is no integer a such that $a^2 - 3a - 19$ is divisible by 289. [15]
3. Show that $3^{2008} + 4^{2009}$ can be written as product of two positive integers each of which is larger than 2009^{182} . [16]
4. Find the sum of all 3-digit natural numbers which contain at least one odd digit and at least one even digit. [15]
5. A convex polygon Γ is such that the distance between any two vertices of Γ does not exceed 1.
 - (i) Prove that the distance between any two points on the boundary of Γ does not exceed 1.
 - (ii) If X and Y are two distinct points inside Γ , prove that there exists a point Z on the boundary of Γ such that $XZ + YZ \leq 1$. [19]
6. In a book with page numbers from 1 to 100, some pages are torn off. The sum of the numbers on the remaining pages is 4949. How many pages are torn off? [19]